Information about conditional statements (Geometry Handbook, p. 12):

Type of Conditional Statement	Example Statement is:
Original Statement: **If *p*, then *q*. ($p \to q$)** • Example: If a number is divisible by 6, then it is divisible by 3. • The original statement may be either true or false.	TRUE
Converse Statement: **If *q*, then *p*. ($q \to p$)** • Example: If a number is divisible by 3, then it is divisible by 6. • The converse statement may be either true or false, and this does not depend on whether the original statement is true or false.	FALSE
Inverse Statement: **If not *p*, then not *q*. ($\sim p \to \sim q$)** • Example: If a number is not divisible by 6, then it is not divisible by 3. • The inverse statement is always true when the converse is true and false when the converse is false.	FALSE
Contrapositive Statement: **If not *q*, then not *p*. ($\sim q \to \sim p$)** • Example: If a number is not divisible by 3, then it is not divisible by 6. • The Contrapositive statement is always true when the original statement is true and false when the original statement is false.	TRUE

Use the following conditional statement for #1 – 2: *If a number is even, then it is divisible by 2.*

p: a number is even

q: it is divisible by 2

1) Write the contrapositive.

If not q, then not p.

If a number is not divisible by 2, then it is not even.

2) Write the inverse.

If not p, then not q.

If a number is not even, then it is not divisible by 2.

Use the following conditional statement for #3 – 5: *If you live in Reno, then you do not live in Colorado.*

p: you live in Reno

q: you do not live in Colorado

3) Write the inverse.

If not p, then not q.

If you do not live in Reno, then you live in Colorado.

4) Write the contrapositive.

If not q, then not p.

If you live in Colorado, then you do not live in Reno.

5) Is the inverse true or false? If false, provide a counter example.

Inverse: If you do not live in Reno, then you do live in Colorado.

The inverse of a true statement may be true or false. In this case it is **false**.

Counter example: If you live in Alabama, then you do not live in Reno, and you also do not live in Colorado.

6) Is the biconditional statement true or false? Explain your reasoning.
Two angles are congruent if and only if they are right angles.

Statement: If two angles are right angles, then they are congruent. This is true because all right angles measure $90°$.

Converse Statement: If two angles are congruent, then they are right angles. This is false because two $45°$ angles, for example, are congruent, but are not right angles.

Therefore, the biconditional statement is false.

Note: if either the original statement or its converse are false, then the bidirectional statement is false.

7) Given: $\frac{3x-1}{5} - 4 = 0$, Prove: $x = 7$.

Step	Statement	Reason
1	$\dfrac{3x-1}{5} - 4 = 0$	Given
2	$\dfrac{3x-1}{5} = 4$	Addition property of equality
3	$3x - 1 = 20$	Multiplication property of equality
4	$3x = 21$	Addition property of equality
5	$x = 7$	Division property of equality

8) Find the complement of the supplement of the angle that is vertical to an angle with a measure of 151°.

Like they say on the dance floor, let's "break it down". Start from the end of the statement and work back toward its beginning.

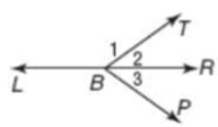
An angle that is vertical to another has the same measure: $151°$.

The supplement of this angle is: $180° - 151° = 29°$

The complement of this angle is: $90° - 29° = \mathbf{61}°$

9) Given: $m\angle 1 + m\angle 3 = 180°$

Prove: $\angle 2 \cong \angle 3$.



Step	Statement	Reason
1	$m\angle 1 + m\angle 3 = 180°$	Given
2	$\angle 1$ and $\angle 3$ are supplementary.	If the sum of two angles is 180°, then the angles are supplementary.
3	$\angle 1$ and $\angle 2$ form a linear pair.	Diagram
4	$\angle 1$ and $\angle 2$ are supplementary.	If two angles form a linear pair, then the angles are supplementary.
5	$\angle 2 \cong \angle 3$	If two angles are supplementary to the same angle, then they are congruent.

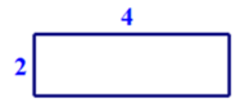
10) Points P, Q, and R are collinear. If PQ = 3, RQ = 15, and PR = 12, which point is between the other two?

Notice that $PQ + PR = RQ$. On the left side of this equation, the point $\boldsymbol{P}$ occurs twice. Therefore, **point $\boldsymbol{P}$** must be the point between the other two.

For #11 – 12: Determine if each statement is true or false. If false, provide a counterexample.

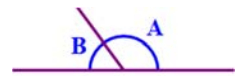
11. If a figure is a rectangle, then it is a square.

False. The figure shown to the right is a rectangle, but is not a square.



12. If two angles are supplementary, then they are both acute angles.

False. In the figure to the right, angles **A** and **B** are supplementary, and angle A is not acute.



For #13 – 17, decide whether the statement is always (A), sometimes (S), or never (N) true. Draw a diagram or explain your reasoning for your work. Work must be shown!

13. If two angles are vertical, then they are congruent.

A – By theorem: if two angles are vertical angles, then the angles are congruent.

14. Vertical angles are complementary.

S – In order for two vertical angles to be complementary, they must each measure $45°$ ($2 \cdot 45° = 90°$). If the vertical angles have any other measure, they are not complementary.

15. Two planes intersect at one point.

N – Two planes must either be parallel (no intersection) or must intersect in a line.

16. If two angles are complementary, then they are both acute angles.

A – For two angles to be complementary, their measures must add to $90°$, so the measure of each must be less $90°$, and so they are both acute.

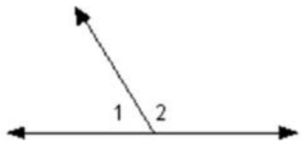
Note: We must assume that both angles are greater than zero for this to be always true. In the odd case of angles that measure $0°$ and $90°$, the statement is not true because a $90°$ angle is not acute. That would change the answer to "Sometimes."

17. If two angles are supplementary, then they form a linear pair.

S – In order for two supplementary angles to form a linear pair, they must be adjacent, i.e., they must share a common ray on one side. Angles do not need to be adjacent to be supplementary.

For #18 – 22: Find the requested values.

18. The ratio of $\angle 1$ to $\angle 2$ is 2:7.
Find $\angle 1$ and $\angle 2$.

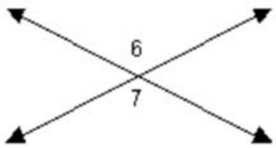


$$m\angle 1 + m\angle 2 = 180°$$

$$\boldsymbol{m\angle 1} = \frac{2}{2+7} \cdot 180° = \mathbf{40°}$$

$$\boldsymbol{m\angle 2} = 180° - m\angle 1 = 180° - 40° = \mathbf{140°}$$

19. $m\angle 6 = x^2$
$m\angle 7 = 5x + 24$
Find x and $\angle 7$.



Vertical angles are congruent, so $m\angle 6 + m\angle 7$.

$$x^2 = 5x + 24$$

$$x^2 - 5x - 24 = 0$$

$$(x-8)(x+3) = 0, \quad \text{so } \boldsymbol{x = -3°, 8°}$$

$m\angle 7 = 5x + 24$. Substituting x-values of $-3°$ and $8°$ in this equation gives:

$$m\angle 7 = (5(-3) + 24)° = \mathbf{9°}$$

$$m\angle 7 = (5(8) + 24)° = \mathbf{64°}$$

Therefore, $\boldsymbol{m\angle 7 = 9°, 64°}$

Note: both solutions for x are valid because they both result in positive angles that are less than 180°. If a solution results in one or more negative angles or an angle greater than 180°, it would have to be discarded.

20. The measure of $\angle F$ is 27° and $\angle G$ is complementary to $\angle F$. $\angle$G is complementary to $\angle$E. Find the measure of $\angle$E.

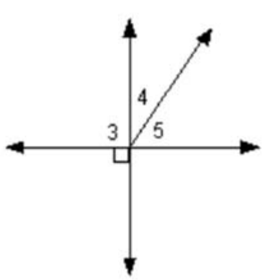
English is confusing. Let's take this one step at a time. Start from the beginning of the statement of the problem.

$$m\angle F = 27°$$

$m\angle G = 90° - 27° = 63°$ Angles G and F are complementary.

$\boldsymbol{m\angle E} = 90° - 63° = \mathbf{27°}$ Angles G and E are complementary.

21. $m\angle 4 = x$
$m\angle 5 = x - 6$
Find x and $\angle 5$.



$$m\angle 4 + m\angle 5 = 90°$$

$$x + (x - 6°) = 90°$$

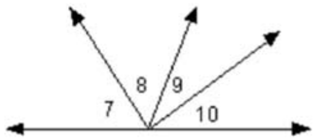
$$2x - 6° = 90°$$

$$2x = 96°$$

$$\boldsymbol{x = 48°}$$

$$\boldsymbol{m\angle 5} = x - 6° = 48° - 6° = \mathbf{42°}$$

22. $\angle 7$ and $\angle 10$ are complementary.
$\angle 8$ and $\angle 10$ are complementary. $m\angle 8 = 25$.
Find $\angle 9$ and $\angle 10$.



$m\angle 7 + m\angle 8 + m\angle 9 + m\angle 10 = 180°$ (diagram)

$m\angle 7 + m\angle 10 = 90°$ (complementary angles)

$m\angle 8 + m\angle 10 = 90°$ (complementary angles)

$m\angle 8 = 25°$ (given)

Calculations:

$m\angle 10 = 90° - m\angle 8 = 90° - 25° = 65°$

$m\angle 7 = m\angle 8 = 25°$ (both complementary to $\angle 10$)

$m\angle 9 = 180° - m\angle 7 - m\angle 8 - m\angle 10$

$= 180° - 25° - 25° - 65° = 65°$

Therefore, $\boldsymbol{m\angle 9 = 65°}$, $\boldsymbol{m\angle 10 = 65°}$

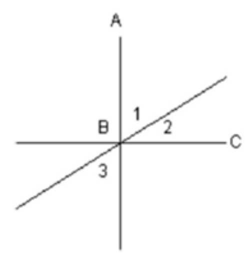
Important note on Problem 22. Notice that in the drawing, angles 9 and 10 do not look like they are 65°. This brings up an important point in doing geometry problems. "You cannot always trust the details in any diagram."

23. Given: $\angle Y$ is supp to $\angle X$, and $\angle Z$ is supp to $\angle X$.
If $\angle Y = 42°$, find the complement of $\angle Z$.

$\angle Y$ and $\angle Z$ are both supplementary to $\angle X$, so $m\angle Z = m\angle Y = 42°$

The complement of $\angle Z$ is: $90° - 42° = \mathbf{48°}$

24. If $\overline{AB} \perp \overline{BC}$, $\angle 1 = (3x + 4y)°$, $\angle 2 = (2x + 2y)°$,
$\angle 3 = (6y - 2)°$, then find $m\angle 3$.



Angles 1 and 2 are complementary because $\overline{AB} \perp \overline{BC}$.

Angles 1 and 3 are congruent because they are vertical angles.

This gives us:

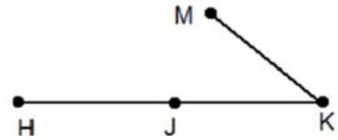
$m\angle 1 + m\angle 2 = 90°$	$m\angle 1 = m\angle 3$
$(3x + 4y) + (2x + 2y) = 90$	$3x + 4y = 6y - 2$
$5x + 6y = 90$	$3x - 2y = -2$

Let's work with these simultaneous equations:

$$5x + 6y = 90 \longrightarrow (\cdot\ 3) \longrightarrow \quad 15x + 18y = 270$$
$$3x - 2y = -2 \longrightarrow (\cdot -5) \longrightarrow \quad + \ -15x + 10y = 10$$
$$28y = 280$$
$$y = 10$$

$\boldsymbol{m\angle 3} = (6y - 2)° = (6(10) - 2)° = (60 - 2)° = \mathbf{58°}$

25. Given: $\overline{KJ} \cong \overline{MK}$
J is the midpoint of $\overline{HK}$.
Prove: $\overline{HJ} \cong \overline{MK}$

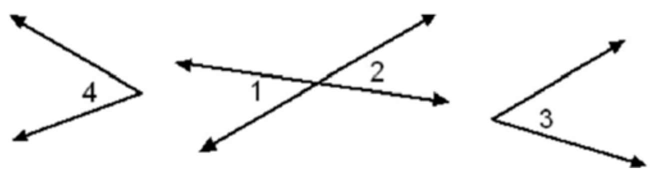


Thought process. Based on the givens, it appears that the three segments identified in the diagram are all congruent. That is, $\overline{HJ} \cong \overline{KJ} \cong \overline{MK}$. We need to work from the congruence we are given to the one we want to prove by considering how the segments relate to each other one pair at a time.

Step	Statement	Reason
1	$\overline{KJ} \cong \overline{MK}$ J is the midpoint of $\overline{HK}$	Given
2	$\overline{KJ} \cong \overline{HJ}$	A midpoint creates two congruent segments.
3	$\overline{HJ} \cong \overline{MK}$	Transitive property of congruence (in this case, two segments that are each congruent to a third segment are congruent to each other).

Note: blue text in the proof is explanatory and is not required to complete the proof.

26. **Given:** $\angle 2$ is complementary to $\angle 3$
$\angle 1$ is complementary to $\angle 4$
Prove: $\angle 4 \cong \angle 3$.



Thought process. We are asked to prove that angles 3 and 4 are congruent. We are given that angles 3 and 4 are complementary to other angles. We should see how those other angles, 1 and 2, relate to each other. Then, we can bring the complementary nature of the given angles into play and see where that leaves us.

Step	Statement	Reason
1	$\angle 2$ and $\angle 3$ are complementary. $\angle 1$ and $\angle 4$ are complementary.	Given
2	$\angle 1$ and $\angle 2$ are vertical angles.	Diagram
3	$\angle 1 \cong \angle 2$	Vertical angles are congruent.
4	$\angle 4 \cong \angle 3$	If two angles are complements of congruent angles, then they are congruent.

27. **Given:** $3x + 16 = 2(x + 6)$

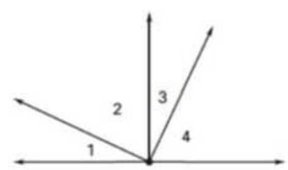
Prove: $x = -4$

Thought process. This is an Algebra problem of the type faced by the student on a regular basis. The problem should be solved as it is in an Algebra class, with the added proviso that we need to justify each step. This is done by identifying the properties of equality used in each step.

Step	Statement	Reason
1	$3x + 16 = 2(x + 6)$	Given
2	$3x + 16 = 2x + 12$	Distributive property of equality
3	$3x = 2x - 4$	Subtraction property of equality (-16)
4	$x = -4$	Subtraction property of equality $(-2x)$

Note: blue text in the proof is explanatory and is not required to complete the proof.

28. Given: $\angle 1$ comp to $\angle 2$
$\angle 1 \cong \angle 3$
$\angle 2 \cong \angle 4$
Prove: $\angle 3$ comp to $\angle 4$



Thought process. We are given a lot of information in this problem, including congruences between pairs of angles. Fortunately, congruences allow substitution, so this suggests we investigate what substitution does for us.

Step	Statement	Reason
1	$\angle 1$ is complementary to $\angle 2$. $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	Given
2	$\angle 3$ is complementary to $\angle 4$.	Substitution of congruent angles ($\angle 3$ for $\angle 1, \angle 4$ for $\angle 2$)

Note: blue text in the proof is explanatory and is not required to complete the proof.

Comment: Proofs are an important part of all higher-level mathematics. The student should work hard to master the skills required to think logically though a problem and to validate each step in problems involving proofs.